**Common Core Standard:**
7.EE.4.b

# Inequalities

## Getting the Idea

An **inequality** is a mathematical sentence that compares two expressions using one of these symbols:

| Inequality | Meaning |
|---|---|
| $<$ | less than |
| $>$ | greater than |
| $\leq$ | less than or equal to |
| $\geq$ | greater than or equal to |

To solve an inequality, follow the same rules as when solving an equation. The solution of an inequality is a set of numbers called the **solution set**.

## Example 1

What is the solution set for this inequality?

$2x - 8 < 11$

**Strategy** **Use inverse operations to isolate the variable.**

**Step 1** Add 8 to both sides of the inequality.

$$2x - 8 < 11$$
$$2x - 8 + 8 < 11 + 8$$
$$2x < 19$$

**Step 2** Divide both sides of the inequality by 2.

$$2x < 19$$
$$\frac{2x}{2} < \frac{19}{2}$$
$$x < \frac{19}{2}$$

**Step 3** Write the improper fraction as a mixed number in simplest form.

$$x < \frac{19}{2}$$
$$x < 9\frac{1}{2}$$

**Solution** $\boldsymbol{x < 9\frac{1}{2}}$

In Example 1, the solution set is all numbers less than $9\frac{1}{2}$. You can check if the solution is correct by testing a value in the original inequality.

Try $\frac{1}{2}$ since $\frac{1}{2} < 9\frac{1}{2}$.

$$2x - 8 < 11$$

$$2\left(\frac{1}{2}\right) - 8 \stackrel{?}{<} 11$$

$$1 - 8 \stackrel{?}{<} 11$$

$$-7 < 11 \quad \longleftarrow \text{ The inequality is true, so the solution is correct.}$$

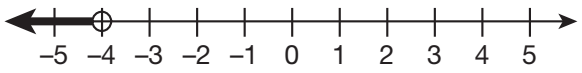
You can often use an inequality to describe a real-world situation.

Use $>$ when you read, "more than." Use $<$ for "less than" or "fewer than."

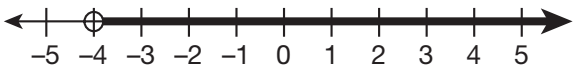
The phrase "at most" means "less than or equal to," so use $\leq$.

The phrase "at least" means "greater than or equal to," so use $\geq$.

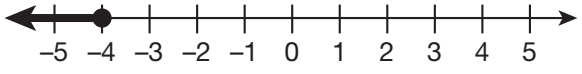
You can graph the solution set of an inequality on a number line. Draw a circle and an arrow to show all of the numbers that are part of the solution. If the circled number is part of the solution set, fill in the circle. If the circled number is not part of the solution set, leave the circle open. Here are some examples.




$x < -4$




$x > -4$




$x \leq -4$


$x \geq -4$

## Example 2

On Saturday, Maya read at least 5 fewer than 3 times as many pages in her book as she did on Friday. On Saturday, she read 58 pages. How many pages did Maya read on Friday? Graph the solution set.

**Strategy** **Translate the problem into an algebraic inequality. Then solve.**

**Step 1** Translate the problem into an algebraic inequality.

Let $n$ represent the number of pages Maya read on Friday.

"5 fewer than 3 times as many pages" translates to the expression $3n - 5$.

Use the inequality sign $\geq$ for "at least."

$3n - 5 \geq 58$

**Step 2** Solve the inequality using inverse operations.

$$3n - 5 \geq 58$$

$3n - 5 + 5 \geq 58 + 5$ Add 5 to both sides.

$$3n \geq 63$$

$\frac{3n}{3} \geq \frac{63}{3}$ Divide both sides by 3.

$$n \geq 21$$

**Step 3** Test the solution in the original inequality.

Try 25, since $25 \geq 21$.

$3n - 5 \geq 58$

$3(25) - 5 \stackrel{?}{\geq} 58$

$75 - 5 \stackrel{?}{\geq} 58$

$70 \geq 58$ ← The inequality is true, so the solution is correct.

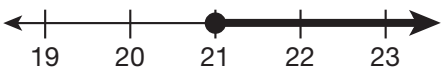
**Step 4** Interpret the solution.

The solution set $n \geq 21$ means Maya read 21 pages or more on Friday.

**Step 5** Graph the solution set $n \geq 21$.

Since 21 is part of the solution, draw a filled-in circle.

Since the inequality sign is $\geq$, draw an arrow pointing to the right.




**Solution** **Maya read at least 21 pages on Friday. The graph is shown in Step 5.**

## Example 3

A taxi driver charges a flat fee of \$4 plus \$6 per mile. The tip is included in the mileage rate. Orlando only has \$22 to pay for a taxi ride. How many miles at most can Orlando ride in the taxi? Graph the solution set.

**Strategy** **Translate the problem into an algebraic inequality. Then solve.**

**Step 1** Translate the problem into an algebraic inequality.

Let $m$ represent the number of miles.

"Flat fee of \$4 plus \$6 per mile" translates to $4 + 6m$.

Use the inequality sign $\leq$ for "at most."

$4 + 6m \leq 22$

**Step 2** Solve the inequality using inverse operations.

$$4 + 6m \leq 22$$

$4 - 4 + 6m \leq 22 - 4$ Subtract 4 from both sides.

$$6m \leq 18$$

$\frac{6m}{6} \leq \frac{18}{6}$ Divide both sides by 6.

$$m \leq 3$$

**Step 3** Test the solution.

Try 2, since $2 \leq 3$.

$4 + 6m \leq 22$

$4 + 6(2) \stackrel{?}{\leq} 22$

$4 + 12 \stackrel{?}{\leq} 22$

$16 \stackrel{?}{\leq} 22$ ← The inequality is true, so the solution is correct.

**Step 4** Interpret the solution.

The solution set $m \leq 3$ means Orlando can ride 3 miles or less.

**Step 5** Graph the solution set.

The graph must start at 0 since he cannot ride less than 0 miles.

Orlando can ride between 0 and 3 miles. So, draw closed circles on 0 and 3.


**Solution** **Orlando can ride between 0 and 3 miles in the taxi.**

## Coached Example

**A middle school is sponsoring a 5K Fun Run to raise money for the library. Each runner will receive a T-shirt. The T-shirts for the race cost $345. Timers and other race equipment cost $85. Local businesses donated $50. If each runner pays $15, how many people must enter the race for it to make a profit?**

The expenses for the race are the _____, the timers, and other race equipment.

The total expenses are _____ + _____ = _____.

Let *n* represent the number of people _____.

Write an expression for the total amount raised by entry fees. _____

How much money was donated to help with expenses? _____

Write an expression for the total amount of money raised. _____

To make a profit, the amount raised by the entry fees plus the money donated must be _____ the total expenses for the race.

Write an inequality to show how many people must enter the race to make a profit.

_____

Solve the inequality.

Test the solution in the original inequality.

The solution of the inequality is a mixed number. Since you cannot have a fraction of a person, round the number up to the next whole number.

What is the least number of people who can enter for the race to make a profit? _____

**At least _____ people must enter the race for it to make a profit.**

## Lesson Practice

**Choose the correct answer.**

**1.** A waitress earned $7 per hour at her job plus an additional $50 in tips on Friday. She earned more than $99 total. Which inequality best represents the situation, where $h$ represents the number of hours she worked on Friday?

**A.** $7 + 50h > 99$
**B.** $7 + 50h \geq 99$
**C.** $7h + 50 > 99$
**D.** $7h + 50 \geq 99$

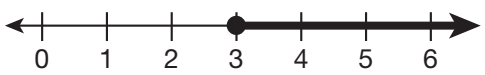
**2.** Which is the solution set of this inequality?

$$2n - 7 < 25$$

**A.** $n > 16$
**B.** $n < 16$
**C.** $n > 9$
**D.** $n < 9$

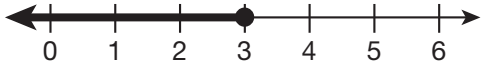
**3.** Which graph represents the solution set of this inequality?

$$5y + 9 < 24$$

**A.**



**B.**



**C.**



**D.**



**4.** Josef has at least 296 photos. He bought an album that holds 6 photos on each page. Which best describes the number of photo album pages he will use?

**A.** at least 50 pages
**B.** more than 50 pages
**C.** less than 50 pages
**D.** at most 50 pages

**5.** Margo has $80 to spend. She wants to buy as many DVDs as possible after buying a CD that costs $8. The DVDs cost $18 each at most. What is the greatest number of DVDs she can buy?

**A.** 3
**B.** 4
**C.** 5
**D.** 6

**6.** Cory earns $20 per day plus $6 for every sale he makes. On Friday, he wants to earn at least $50. Which best describes the number of sales he needs to make to reach his goal?

**A.** at least 5 sales
**B.** at most 5 sales
**C.** at least 6 sales
**D.** at most 6 sales

**7.** Sharona's age is at most 3 more than twice Kayla's age. If Sharona is 35 years old, which inequality best represents the situation, where $a$ represents Kayla's age?

**A.** $2a + 3 < 35$

**B.** $2a + 3 \leq 35$

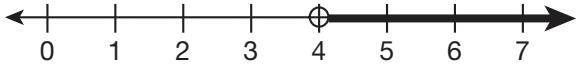
**C.** $2a + 3 > 35$

**D.** $2a + 3 \geq 35$

**8.** Which graph represents the solution set to this inequality?

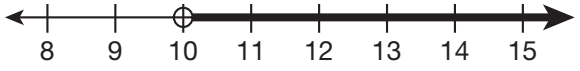
$3x + 9 \geq 21$

**A.**

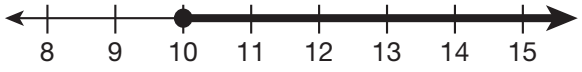



**B.**


**C.**




**D.**




**9.** The soccer team is selling key chains to raise money for the team. They ordered 100 key chains that cost \$0.25 each. There is a flat shipping rate of \$8. The team sells the key chains for \$2 each.

**A.** Write an algebraic inequality to find the fewest number of key chains, $k$, the team must sell to make a profit.

**B.** What is the fewest number of key chains the team must sell to make a profit? Show your work and explain your thinking.

**10.** Which symbol does each of the given words indicate to use in an inequality? Circle each correct symbol.

| | | | | | |
|---|---|---|---|---|---|
| At most: | $<$ | Less than: | $<$ | At least: | $>$ |
| | $\leq$ | | $\leq$ | | $\leq$ |
| | $\geq$ | | $>$ | | $\geq$ |

**11.** Landon's baseball practice lasts more than twice as long as math class. Select True or False for each statement.

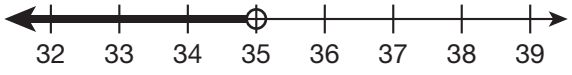
**A.** If math class lasts 52 minutes, then baseball practice could last 95 minutes. ○ True ○ False

**B.** If math class lasts 45 minutes, then baseball practice could last 90 minutes. ○ True ○ False

**C.** If math class lasts 65 minutes, then baseball practice could last 45 minutes. ○ True ○ False

**D.** If math class lasts 57 minutes, then baseball practice could last 120 minutes. ○ True ○ False

**12.** Corey graphed a solution set to a problem. Which value is part of the solution set? Circle all that apply.




**A.** 17
**B.** 37
**C.** 34.5
**D.** 0
**E.** 35
**F.** 35.5
**G.** 40

**13.** Caitlin has $80 and earns $7.25 an hour babysitting. To take a creative writing course during the summer, she needs to have at least $250. How many hours must she babysit to cover the cost of the writing course? Use numbers from the box to complete the inequalities and solve the problem.

_____ + _____ $h \geq 250$

_____ $h \geq$ _____

$h \geq$ _____

Caitlin has to babysit _____ hours.

| | |
|---|---|
| 7.25 | 46 |
| 23.4 | 80 |
| 24 | 170 |
| 45.5 | 330 |

**14.** Payson filled a 10-gallon jug by using a smaller container. She needed to fill the container at most 18 times. Could each measure be the size of the container? Select Yes or No.

**A.** 1 gal ○ Yes ○ No

**B.** 2 gal ○ Yes ○ No

**C.** $\frac{1}{2}$ gal ○ Yes ○ No

**D.** $\frac{3}{4}$ gal ○ Yes ○ No

**15.** Sean has at most $50 to spend while shopping. He wants to buy some books that cost $4 each and a DVD that cost $17. What is the greatest number of books he can buy? Use numbers from the box to complete the inequalities and solve the problem.

_____ $x$ + _____ $\leq 50$

_____ $x \leq$ _____

$x \leq$ _____

Sean can buy _____ books.

| | |
|---|---|
| 4 | 16.25 |
| 8 | 17 |
| 8.25 | 33 |
| 16 | 67 |